

Detection of non-self-correcting nature of information cascade

Shintaro Mori^{1**}, Masafumi Hino², Masato Hisakado³, and Taiki Takahashi^{4,5}

¹ Department of Physics, Kitasato University
1-15-1, Kitasato, Sagami-hara, Kanagawa 252-0373, Japan

² NEC Corporation
Siba 5-7-1, Minato-ku, Tokyo 108-8001, Japan

³ Financial Services Agency
Kasumigaseki 3-2-1, Chiyoda-ku, Tokyo 100-8967, Japan

⁴ Department of Behavioral Science, Faculty of Letters

⁵ Center for Experimental Research in Social Sciences, Hokkaido University
Kita 10, Nishi 7, Kita-ku, Sapporo, Hokkaido 060-0810, Japan

Abstract. We propose a method of detecting non-self-correcting information cascades in experiments in which subjects choose an option sequentially by observing the choices of previous subjects. The method uses the correlation function $C(t)$ between the first and the $t + 1$ -th subject's choices. $C(t)$ measures the strength of the domino effect, and the limit value $c \equiv \lim_{t \rightarrow \infty} C(t)$ determines whether the domino effect lasts forever ($c > 0$) or not ($c = 0$). The condition $c > 0$ is an adequate condition for a non-self-correcting system, and the probability that the majority's choice remains wrong in the limit $t \rightarrow \infty$ is positive. We apply the method to data from two experiments in which T subjects answered two-choice questions: (i) general knowledge questions ($T_{avg} = 60$) and (ii) urn-choice questions ($T = 63$). We find $c > 0$ for difficult questions in (i) and all cases in (ii), and the systems are not self-correcting.

Keywords: information cascade-herding-self-correcting

1 Introduction

Herding phenomena are ubiquitous in human and animal behavior [1,2]. An example is an information cascade, in which a person observes others' choices and chooses the majority's choice even though the person's private signal contradicts it [3,4]. It is a rational behavior for people who are uncertain about choosing. If an information cascade occurs, the same mechanism applies to later decision-makers, and the majority's choice tends to prevail. In some cases, the successive choices are wrong, and the cascade leads to irrational herding behavior [5].

An experimental setup demonstrates a situation in which an information cascade occurs [6]. There are two urns, A and B, and urn A (B) contains two a (b) balls and one b (a) ball. In each run of the experiment, an urn is randomly

^{**} corresponding author, mori@sci.kitasato-u.ac.jp

chosen initially and called X. Then, the subjects guess whether urn X is A or B and choose sequentially. They get a reward for the correct choice. In the course of the experiment, each subject draws a ball from X, which is his private signal. If the ball is a (b), urn X is more likely to be A (B). He also observes the choices of the previous subjects. If the difference between the numbers of subjects who choose each urn exceeds two, the private signal cannot overcome the majority's choice. An information cascade starts if someone chooses the majority's choice although his private signal suggests the minority's one. As the probability that the first two persons both choose the wrong option is non-zero, the probability for the onset of a cascade where the majority's choice is wrong is positive.

We now consider whether the wrong cascade continues [5]. If it continues forever, the majority's choice converges to the wrong option. Information cascades were initially considered to be fragile phenomena. As the trigger of the cascade is a small imbalance, people can be dissuaded from following the majority's choice [3]. In addition, an agent model with a Bayesian update of the private belief showed that the information cascade is self-correcting [8]. As the number of agents tends toward infinity, the wrong cascade disappears, and the majority's choice converges to the optimal option.

Using an information cascade experiment with a general knowledge two-choice quiz, we have shown that a phase transition occurs between a one-peak phase and a two-peak phase [9]. If the questions are easy, the ratio $z(t)$ of the correct choices of t subjects converges to a value $z_+ > 1/2$ in the limit $t \rightarrow \infty$. As there is only one peak in the probability distribution function of $z(t)$, we call the corresponding phase the one-peak phase [10,11]. If the questions are difficult and most people do not know the answers, $z(t)$ converges to $z_+ > 1/2$ or $z_- < 1/2$. One cannot predict the value in $\{z_+, z_-\}$ to which $z(t)$ converges. We call the corresponding phase the two-peak phase. In the two-peak phase, the wrong cascade does not necessarily disappear, and the system is not self-correcting.

It was recently shown that the limit value of the normalized correlation function is the order parameter of the phase transition [14]. The normalized correlation function shows how the first subject's choice propagates to later subjects. It provides a measure of the domino effect. In addition, the positiveness of the limit value is a sufficient condition for a non-self-correcting system. By extrapolating the results for a finite system to infinity, we can determine whether the system is self-correcting. We report on the application of the method to data from two types of information cascade experiments. In section 2, we define the normalized correlation function. We also explain the behavior of the function in each phase and the extrapolation method used to estimate its limit. We present the results of the data analysis in section 3. Section 4 summarizes the results.

2 Correlation function and asymptotic behaviors

We consider a typical information cascade experiment. T subjects answer a two-choice question sequentially in each run. We denote the order of the subjects as

t , where $t = 1, 2, \dots, T$. We denote the choice of subject t by $X(t) \in \{0, 1\}$, $t = 1, 2, \dots, T$. If the choice is true (false), $X(t)$ takes 1 (0).

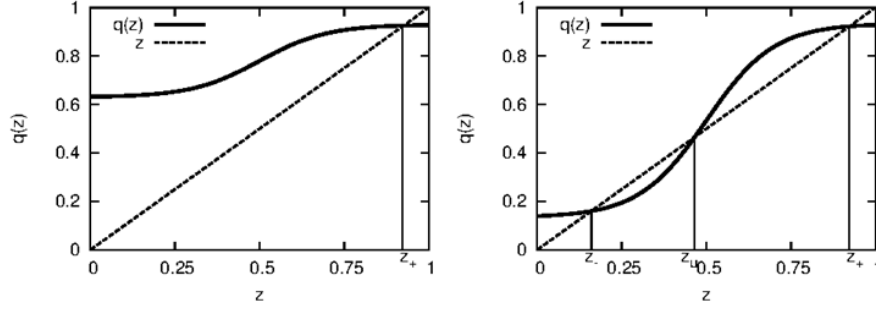


Fig. 1. Response function $q(z)$ vs. z . Left panel shows the one-peak phase, in which there is one solution, z_+ , for $z = q(z)$. Right panel shows the two-peak phase, in which there are three solutions, $z_- < z_u < z_+$, for $z = q(z)$.

The correlation function $C(t)$ is defined as the covariance between $X(1)$ and $X(t+1)$ divided by the variance of $X(1)$:

$$C(t) \equiv \text{Cov}(X(1), X(t+1)) / \text{Var}(X(1)).$$

$C(t)$ can be expressed as the difference of two conditional probabilities.

$$C(t) = \Pr(X(t+1) = 1 | X(1) = 1) - \Pr(X(t+1) = 1 | X(1) = 0). \quad (1)$$

$C(t)$ shows the degree to which the first subject's choice is transmitted to later subjects. It is a measure of the domino effect in an information cascade.

$C(t)$ is generally positive, and its asymptotic behavior depends on the phase of the system and the shape of the response function $q(z)$. Here $q(z)$ represents the dependence of the probability of the correct choice by subject $t+1$ on the ratio $z(t)$ of the correct choices of the previous t subjects.

$$q(z) \equiv \Pr(X(t+1) = 1 | z(t) = z), \quad z(t) = \frac{1}{t} \sum_{s=1}^t X(s).$$

With the definition of $q(z)$, the stochastic process $\{X(t)\}, t = 1, 2, \dots$ becomes a generalized Pólya urn process [12]. If there is one solution for $z = q(z)$ at z_+ (left panel in Fig.1), $z(t)$ converges to z_+ . $C(t)$ shows power-law decay for large t with two constants, c' and l , as

$$C(t) \simeq c' \cdot t^{l-1} \quad l < 1.$$

Here, l is the exponent for the power-law decay and is less than 1. The value of l is given by $g'(z_+)$ [13,11]. If there are three solutions for $z = q(z)$ at

$z_- < z_u < z_+$ (right panel in Fig.1), the system is in the two-peak phase; $\lim_{t \rightarrow \infty} z(t) = z_+$ or z_- [12]. The limit value $c \equiv \lim_{t \rightarrow \infty} C(t)$ is positive, and the first subject's choice propagates to an infinite number of later subjects [14]. $C(t)$ behaves asymptotically as

$$C(t) \sim c + c' \cdot t^{l-1}. \quad (2)$$

Here $c' \cdot t^{l-1}$ is the subleading term of $C(t)$, and l is given by the larger value among $\{g'(z_+), g'(z_-)\}$. Further, c acts as an order parameter of the phase transition, and eq.(2) is the general asymptotic behavior of $C(t)$ [15].

As it is difficult to estimate c using $c \equiv \lim_{t \rightarrow \infty} C(t)$ with empirical data, where the system size and number of samples are strictly limited, we introduce two quantities for the estimation. First, we define the n -th moment $m_n(t)$ for $C(t)$ as $m_n(t) \equiv \sum_{s=0}^{t-1} C(s)(s/t)^n$. We define the integrated correlation time $\tau(t)$ as $\tau(t) = m_0(t)$. We also define the second moment correlation time $\xi(t)$ as $\xi(t) \equiv t \cdot \sqrt{m_2(t)/m_0(t)}$. Using the asymptotic behavior of $C(t)$, we estimate the subsequent asymptotic behavior of $\tau(t)/t$ and $\xi(t)/t$.

$$\tau(t)/t \simeq c + \frac{c'}{l} \cdot t^{l-1} \quad (3)$$

$$\xi(t)/t \rightarrow \begin{cases} \sqrt{l/l+2} & c = 0 \\ \sqrt{1/3} & c > 0 \end{cases} \quad (4)$$

As $\tau(t)/t$ is defined as the summation of $C(s)$ over $0 \leq s < t$ divided by t , the standard error becomes smaller than that of $C(t)$. The asymptotic behavior of $\tau(t)/t$ in eq.(3) provides a more reliable estimate of c and l than the fitting of $C(t)$ to eq.(2). $\xi(t)/t$ also provides a reliable estimate for l [15]. If $c > 0$, the leading term of $C(t)$ is the constant c , and l should be interpreted as $l = 1$.

We define whether the system is self-correcting according to whether $z(t)$ always converges to z_+ . In the one-peak (two-peak) phase, the system is (non-)self-correcting. If $c > 0$, the system is in the two-peak phase and is non-self-correcting. However, $c = 0$ does not necessarily mean that the system is self-correcting. For the system to be self-correcting, $q(z) = z$ has to have only one solution, z_+ .

3 Domino effect and detection of non-self-correcting nature

We study the domino effect and non-self-correction in information cascades. We discuss two types of information cascade experiments.

In experiment 1 (EXP-I), subjects answered a general knowledge two-choice quiz. First, the subjects answered using only their own knowledge. Then, they observed the choices of previous subjects and answered the question again. The average length of the sequence of subjects is $T = 60$, and the number of choice sequences is 240. The choice sequences are classified into four bins according

to the ratio of correct choices $z_0(T)$ of the first answers without observation as $z_0(T) = 50\% \pm 5\%, 60\% \pm 5\%, 70\% \pm 5\%$, and $80\% \pm 5\%$, and the number of samples in each bin is $38(50\% \pm 5\%), 52(60\% \pm 5\%), 38(70\% \pm 5\%)$, and $38(80\% \pm 5\%)$, respectively [16].

Experiment 2 (EXP-II) is similar to the situation explained in the Introduction. There are two urns, A and B, which contain a and b balls in different configurations. We use two configuration patterns: (i) two a balls and one b ball in urn A vs. one a ball and two b balls in urn B and (ii) five a balls and four b balls in urn A vs. four a balls and five b balls in urn B. Urn $X \in \{A, B\}$ is chosen at random at the beginning of each run, and subjects are asked to choose between A or B. Each subject draws one ball from X and checks whether it is a or b . The ball corresponds to the type of urn X with probability $q = 2/3(5/9)$ for (i) [(ii)]. In addition, the subject also observes the choices of previous subjects. Our results, unlike those of previous experiments [6,7,8], show the summary statistics of the number of subjects who have chosen each urn. The length T and number of questions I are 63 and 200, respectively, for $q \in \{2/3, 5/9\}$ [17].

We denote the choice sequences in each bin as $\{X(i, t)\}, i = 1, \dots, I, t = 1, \dots, T(i)$. Here, the length of the sequence depends on question i in EXP-I; we denote it as $T(i)$. The number of samples I also depends on the bins. In EXP-II, $T(i) = 63$, and $I = 200$. First, we estimate $C(t)$ and its standard error $\Delta C(t)$ using eq.(1). We denote the estimate and standard error of the probabilities as $q_x(t+1) = \Pr(X(t+1) = 1 | X(1) = x)$ and $\Delta q_x(t+1)$, respectively. They are estimated from experimental data $\{X(i, t)\}$ as

$$q_x(t+1) = \frac{1 + \sum_{i=1}^I X(i, t+1) \delta_{X(i,1),x}}{N_x + 2},$$

$$N_x = \sum_{i=1}^I \delta_{X(i,1),x},$$

$$\Delta q_x(t+1) = \sqrt{\frac{q(x, t+1)(1 - q_x(t+1))}{N_x + 3}}.$$

Here, we use the expectation value and standard deviation obtained from the posterior probability distribution for the probabilities. $C(t)$ is then estimated as

$$C(t) = q_1(t+1) - q_0(t+1).$$

The error bars of $C(t)$ are given as

$$\Delta C(t) = \sqrt{\Delta q_1(t+1)^2 + \Delta q_0(t+1)^2}. \quad (5)$$

Using $C(t)$ and $\Delta C(t)$, we estimate the error bars of $m_n(t)$ as

$$\Delta m_n(t) = \sqrt{\sum_{s=1}^{t-1} \Delta C(s)^2 (s/t)^{2n}}.$$

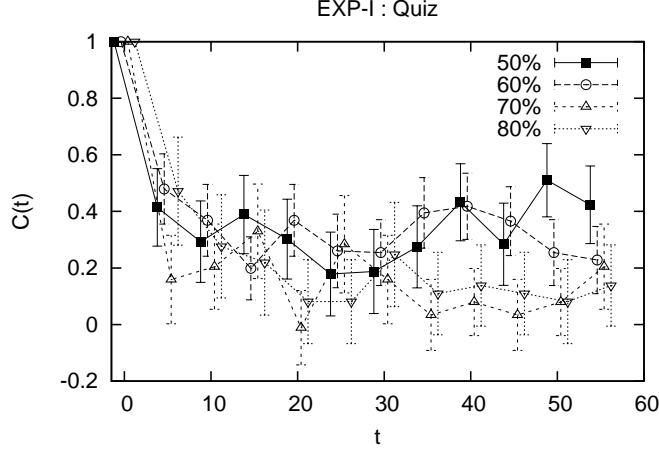


Fig. 2. $C(t)$ vs. t for EXP-I. The sample choice sequences are classified according to the value of $z_0(T)$ as $z_0(T) = 50\% \pm 5\%$ (\blacksquare), $60\% \pm 5\%$ (\circ), $70\% \pm 5\%$ (\triangle), and $80\% \pm 5\%$ (∇). We plot only data with the interval $\Delta t = 5$. To see the behavior clearly, we slightly shift the data horizontally.

Here we assume that $\Delta C(s)$ and $\Delta C(s')$ are independent of each other if $s \neq s'$. We estimate the error bars of $\tau_t(t)$ and $\xi_t(t)$ as

$$\begin{aligned} \Delta\tau_t &= \frac{1}{t} \Delta m_0(t), \\ \Delta\xi_t &= \sqrt{\xi_t} (\Delta m_2(t)/2m_2(t) + \Delta m_0(t)/2m_0(t)). \end{aligned} \quad (6)$$

In the estimation of $\Delta\xi_t$, we assume that $\Delta m_2(t)$ and Δm_0 are completely correlated.

3.1 EXP-I: General knowledge quiz case

Figure 2 plots $C(t)$ vs. t . The value of $C(t)$ generally decreases from its initial value of 1 with increasing t . Because the sample number is restricted, $\Delta C(t)$ is large. We see that for difficult questions with $z_0(T) = 50\% \pm 5\%$ and $60\% \pm 5\%$, $C(t)$ is positive for large values of t . On the other hand, for easy questions with $z_0(T) = 70\% \pm 5\%$ and $80\% \pm 5\%$, $C(t)$ decreases to zero with increasing t . These results suggest that the system is in the two-peak phase for difficult questions. For $z_0(T) = 70\% \pm 5\%$ and $80\% \pm 5\%$, an analysis of $q(z)$ showed that the system was in the one-peak phase [16].

Figure 3 shows plots of $\xi(t)/t$ and $\tau(t)/t$ vs. t . The standard errors for $\xi(t)/t$ are larger than those for $\tau(t)/t$ because $\xi(t)$ is calculated with the second moment $m_2(t)$. For large values of t , $\xi(t)/t$ takes $\sqrt{1/3}$ for difficult questions with $z_0(T) = 50\% \pm 5\%$ and $60\% \pm 5\%$. The results suggest that the system is in the two-peak phase. For easy questions with $z_0(T) = 70\% \pm 5\%$ and $80\% \pm 5\%$, $\xi(t)/t \simeq 0.5$

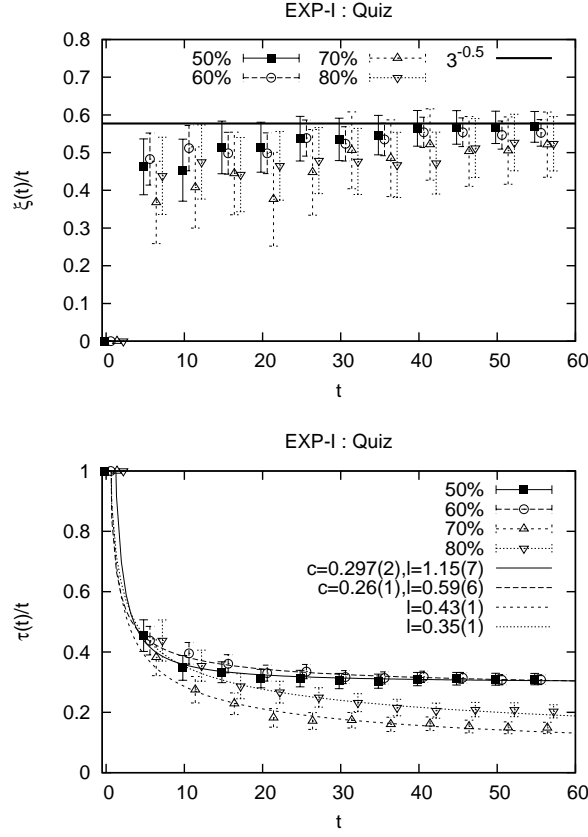


Fig. 3. $\xi(t)/t$ and $\tau(t)/t$ vs. t for EXP-I with the interval $\Delta t = 5$. We also plot the fitted results for $\tau(t)/t$.

for large values of t . As $\xi(t)/t \simeq \sqrt{l/l+2}$, $l \simeq 0.7$ for easy questions. As l is smaller than 1, the system is in the one-peak phase.

As the system is considered to be in the two-peak phase for $z_0(T) = 50\% \pm 5\%$ and $60\% \pm 5\%$, we assume $\tau(t)/t = c + d \cdot t^{l-1}$ and estimate c, l, d using the least square fit. We find that $c = 0.297(2)$ for $z_0(T) = 50\% \pm 5\%$ and $c = 0.26(1)$ for $z_0(T) = 60\% \pm 5\%$. For $z_0(T) = 70\% \pm 5\%$ and $80\% \pm 5\%$, we assume $\tau(t)/t = d \cdot t^{l-1}$ and estimate l and d . We find that $l = 0.43(1)$ for $z_0(T) = 70\% \pm 5\%$ and $l = 0.35(1)$ for $z_0(T) = 80\% \pm 5\%$, which differ slightly from the value of $l \simeq 0.7$ estimated from $\xi(t)/t$.

3.2 EXP-II: Urn choice case

Figure 4 shows plots of $C(t)$, $\xi(t)/t$, and $\tau(t)/t$ vs. t for $q \in \{2/3, 5/9\}$. As the number of samples is larger than that in EXP-I, the standard errors are smaller

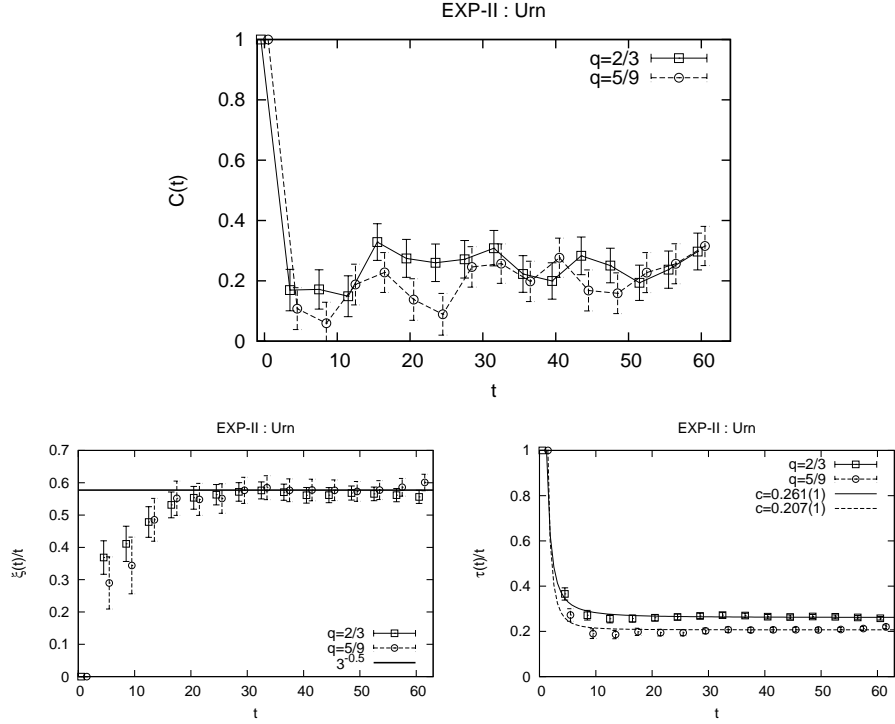


Fig. 4. $C(t)$, $\xi(t)/t$, and $\tau(t)/t$ vs. t for EXP-II. We use the symbol $\square(\bigcirc)$ for $q = 2/3(5/9)$. We plot only data with the interval $\Delta t = 4$. To see the behavior clearly, we slightly shift the data horizontally.

than the symbols' size for $\tau(t)/t$ and large t . We see that $C(t)$ is positive for large values of t for both cases of q , where $q \in \{2/3, 5/9\}$. In addition, $\xi(t)/t$ for large values of t converges to $\sqrt{1/3}$, and the exponent l for $C(t) \sim t^{l-1}$ is almost one. These results suggest that the system is in the two-peak phase for both values of q . We assume $\tau(t)/t = c + d \cdot t^{l-1}$ and estimate c, l, d using the least square fit. We find that $c = 0.261(1)$ for $q = 2/3$ and $c = 0.207(1)$ for $q = 5/9$.

4 Conclusion

We studied the self-correcting nature of information cascades. We proposed the use of the normalized correlation function $C(t)$, which shows how the first subject's choice is propagated to later subjects and measures the strength of the domino effect in information cascades. $c \equiv \lim_{t \rightarrow \infty} C(t) > 0$ is a sufficient condition for a non-self-correcting information cascade. In this case, the domino effect continues infinitely. The system is in the two-peak phase, and the probability

that $z(t)$ converges to $z_- < 1/2$ is positive. We used data from two types of information cascade experiment: EXP-I, which used a general knowledge quiz, and EXP-II, which used urns. The accuracy q of the private signal is $q \in \{2/3, 5/9\}$ in EXP-II. We estimate $C(t)$ and its integrated quantities $\tau(t)$ and $\xi(t)$. In EXP-I, when the questions were difficult, $c > 0$. In EXP-II, $c > 0$ for both cases of q where $q \in \{2/3, 5/9\}$. In these cases, the system is non-self-correcting.

We focus on the study of the non-self-correcting nature of information cascades. Although $c > 0$ is a sufficient condition for a non-self-correcting cascade, $c = 0$ is not a sufficient condition for a self-correcting cascade. To verify this, one should study the response function $q(z)$ and count the number of solutions for $z = q(z)$. Alternatively, it is necessary to study the limit value of the variance of $z(t)$. If there is only one solution, $z_+ > 1/2$, or the limit value is zero, the system is self-correcting. In EXP-I, we studied these points and concluded that the system is self-correcting for $z_0(T) = 70\% \pm 5\%$ and $80\% \pm 5\%$ [16]. Our experiment for EXP-II and its analysis are under way [17].

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